

Real Analysis 1, MATH 5210, Fall 2024

Homework 4, Section 2.2. Lebesgue Outer Measure

Due Saturday, September 21, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation given in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu).

- 2.7.** Prove that for any bounded set E , there is a G_δ set G for which $E \subset G$ and $m^*(G) = m^*(E)$. Set G is called the *measurable cover* of E . See Theorem 3.1 of the supplemental notes to Section 2.3. (In fact, this result also holds if set E is not bounded, as long as it is of finite measure.)
- 2.8.** Let B be the set of rational numbers in the interval $[0, 1]$, and let $\{I_k\}_{k=1}^n$ be a finite collection of open intervals that covers B . Prove that $\sum_{k=1}^n m^*(I_k) > 1$. HINT: Mimic the proof of Proposition 2.1 for intervals of the type $[a, b]$.
- 2.10** Let A and B be bounded sets for which there is an $\alpha > 0$ such that $|a - b| \geq \alpha$ for all $a \in A$ and $b \in B$. Prove that $m^*(A \cup B) = m^*(A) + m^*(B)$.