Real Analysis 1, MATH 5210, Fall 2024

Homework 6, Section 2.4. Outer and Inner Approximation of Lebesgue Measurable Sets

Due Saturday, October 19, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

2.16. (a) Partially complete the proof of Theorem 2.11 by showing (iii): Set E is measurable if and only if for all $\varepsilon > 0$ there is a closed set F with $F \subset E$ for which $m^*(E \setminus F) < \varepsilon$. HINT: E is measurable if and only if E^c is measurable, and $E \setminus \mathcal{O}^c = \mathcal{O} \setminus E^c$. To show that E is measurable given such closed set F, consider the (easily verified) fact that $A \cap E = (A \cap F) \cup (A \cap (E \setminus F))$.

2.19. Let E have finite outer measure. Prove that if E is not measurable, then there is an open set $\mathcal{O}$ containing E that has finite outer measure and for which

$$m^*(\mathcal{O} \setminus E) > m^*(\mathcal{O}) - m^*(E).$$

NOTE: This is our first encounter with the behavior of a (Lebesgue) non-measurable set. It will get weirder.