Real Analysis 1, MATH 5210, Fall 2024

Homework 8, Section 3.1. Sums, Products, and Compositions

Due Saturday, November 9, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

Problem numbers are based on the 5th edition of Royden and Fitzpatrick.

3.3. Provide an example of a function $f : [a, b] \to \mathbb{R}$ that is not measurable, while both $|f|$ and f^2 are measurable functions. HINT: Use set P of Section 2.6 from Royden and Fitzpatrick's 3rd edition. You may assume that set $\frac{1}{b-a}P = \{x \mid (b-a)x \in P\}$ in not measurable.

3.4. Let $\{f_n : E \to \mathbb{R}\}$ be a sequence of measurable functions defined on measurable set E. Define E_0 to be the set of points $x \in E_0$ at which the sequence of real numbers $\{f_n(x)\}_{n=1}^{\infty}$ converges to a real number. Is the set E_0 necessarily measurable? If so prove it, and if not then show this by example.

3.7. Suppose the function f is defined on a measurable set E and suppose f has the property that $\{x \in E \mid f(x) > c\} \in \mathcal{M}$ for each rational number c. Is f necessarily measurable? If so, then prove it. If not, then give an example.