

# Real Analysis 1, MATH 5210, Fall 2024

## Homework 9, Section 3.2. Sequential Pointwise Limits and Simple Approximation

Due Saturday, November 16, at 11:59 p.m.

**Write in complete sentences and paragraphs!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu).

Some problem numbers are based on the 4th edition of Royden and Fitzpatrick and some are based on the 5th edition.

**3.13.** (5th Edition) Let  $\{f_n : [a, b] \rightarrow \mathbb{R}\}_{n=1}^{\infty}$  be an increasing sequence of continuous functions that converge pointwise to the continuous function  $f : [a, b] \rightarrow \mathbb{R}$ . Prove that the convergence is uniform on  $[a, b]$ . HINT: Let  $\varepsilon > 0$ . For each  $n \in \mathbb{N}$  define  $E_n = \{x \in [a, b] \mid |f(x) - f_n(x)| < \varepsilon\}$ . Prove that  $\{E_n\}_{n=1}^{\infty}$  is a cover of  $[a, b]$  with sets open relative to  $[a, b]$  and use the Heine-Borel Theorem.

**3.19.** (5th Edition) For a sequence  $\{f_n\}$  of measurable functions with common domain  $E$ , prove that  $\inf\{f_n\}$  and  $\sup\{f_n\}$  are measurable. HINT: Use the definition of measurable on  $\inf\{f_n\}$  and  $\sup\{f_n\}$ .

**3.19(a).** (4th Edition) Prove that the sum and product of two simple functions are simple.