Real Analysis 2, MATH 5220

Homework 2, Section 6.6

Due Thursday February 5, 2015 at 2:15

Lemma 6.16. Let φ be a convex function on (a, b). Then φ has left-hand and right-hand derivatives at each point $x \in (a, b)$. Moreover, for points $u, v \in (a, b)$ with $u < v$ these one-sided derivatives satisfy the following inequality:

$$\varphi'(u^-) \leq \varphi'(u^+) \leq \frac{\varphi(v) - \varphi(u)}{v - u} \leq \varphi'(v^-) \leq \varphi'(v^+).$$

HINT: Use the Chordal Slope Lemma.

6.66. For what (convex) functions φ is Jensen's Inequality an equality for all integrable f? Prove your claim.

6.67. State and prove a version of Jensen's Inequality on a general closed, bounded interval $[a, b]$. HINT: In the proof of Jensen's Inequality, replace the supporting line at α with a supporting line at $\frac{\alpha}{b-a}$ where $\alpha = \int_a^b f(x)\,dx$.