Real Analysis 2, MATH 5220

Homework 5, Section 11.1 and 11.2

Due Friday March 20, 2015 at 3:00

11.3. For a nonempty set X, let $\mathcal{B}$ be a collection of subsets of X. Suppose $\mathcal{B}$ is a base for a topology on X. Prove that

(i) $\mathcal{B}$ covers X. That is, $X = \cup_{B \in \mathcal{B}} B$.

(ii) If B_1 and B_2 are in $\mathcal{B}$ and $x \in B_1 \cap B_2$, then there is a set B in $\mathcal{B}$ for which $x \in B \subseteq B_1 \cap B_2$.

11.5(i). Let E be a subset of a topological space X. A point $x \in X$ is an *interior point* of E provided there is a neighborhood of x that is contained in E. The collection of interior points of E is the *interior* of E, denoted $\text{int}(E)$. Prove that $\text{int}(E)$ is open if and only if $E = \text{int}(E)$.

11.9. Prove that the collection of all intervals of the form $[a, b)$, where $a < b$, is a base for a topology for the set of real numbers. The real numbers $\mathbb{R}$ with this topology is called the *Sorgenfrey Line*. Prove that the Sorgenfrey line is Hausdorff (see Section 11.2—this part is Exercise 11.6(a)).

11.A. Let $X = [0, 1] \cup \{0'\}$ and consider the sets of the form (α, β), $[0, \gamma)$, $(\delta, 1]$, and $\{0'\} \cup (0, \eta)$ where $\alpha, \beta, \gamma, \delta, \eta \in (0, 1)$. Prove that the sets are a base for a topology on X and that this topology is Tychonoff but not Hausdorff.