Real Analysis 2, MATH 5220

Homework 6, Section 11.3 and 11.4

Due Monday March 30, 2015 at 3:00

11.17. A topological space is said to be a *Lindelöf* space or to have the *Lindelöf property* provided each open cover of X has a countable subcover. Prove that if X is second countable then it is Lindelöf.

11.18. Let X be an uncountable set of points, and let $\mathcal{T}$ consist of $\varnothing$ and all subsets of X that have finite complements. Prove that $\mathcal{T}$ is a topology for X and that the space $(X, \mathcal{T})$ is not first countable.

11.21. Prove that the Sorgenfrey line is first countable but not second countable and yet the rationals are dense. NOTE: By showing the rationals are dense, we have that the Sorgenfrey line is separable. In Proposition 9.25, it is shown that a metric space is separable if and only if the topology induced by the metric is second countable. Since the Sorgenfrey line is separable and not second countable, then it's topology cannot be induced by a metric; that is, it is not metrizable.

11.36. Prove that $\mathbb{R}$ is homeomorphic to the open interval $(0, 1)$ but is not homeomorphic to $[0, 1]$. Assume the usual topology on $\mathbb{R}$ and the resulting induced topology on $(0, 1)$ and $[0, 1]$.