Real Analysis 2, MATH 5220

Homework 7, Section 11.5 and 11.6 (Modified)

Due Tuesday April 7, 2015 at 2:15

11.39. Let topological space $(X, \mathcal{T})$ be second countable. Prove that X is compact if and only if it is countably compact.

11.47. Let $\{C_\lambda\}_{\lambda\in\Lambda}$ be a collection of connected subsets of a topological space X and suppose that any two of them have a point in common. Prove that the union of $\{C_\lambda\}_{\lambda\in\Lambda}$ is connected. HINT: Do by contradiction.

11.48. Let A be a connected subset of a topological space $(X, \mathcal{T})$. Suppose $A \subseteq B \subseteq \overline{A}$. Prove that B is connected. HINT: Do by contradiction.

11.41. **(Bonus)** Let $(X, \mathcal{T})$ be a compact Hausdorff space and $\{F_n\}_{n=1}^{\infty}$ a descending collection of closed subsets of X (that is, $F_{n+1} \subset F_n$). Let $\mathcal{O}$ be a neighborhood of the intersection $\cap_{n=1}^{\infty} F_n$. Prove that there is $N \in \mathbb{N}$ such that $F_n \subset \mathcal{O}$ for all $n \geq N$.