Real Analysis 2, MATH 5220

Homework 8, Munkres Section 22 and 51

Due Wednesday April 15, 2015 at 2:15

22.2(a). Let $p : X \to Y$ be a continuous map. Prove that if there is a continuous map $f : Y \to X$ such that $p \circ f$ equals the identity map of Y, then p is a quotient map.

22.2(b). If $A \subset X$, then a *retraction* of X onto A is a continuous map $r : X \to A$ such that $r(a) = a$ for all $a \in A$. Prove that a retraction is a quotient map. HINT: Use Part (a).

22.4(a,b). (a) Define an equivalence relation on the plane $X = \mathbb{R}^2$ (under the usual topology) as $(x_0, y_0) \sim (x_1, y_1)$ if and only if $x_0 + y_0^2 = x_1 + y_1^2$. Let X^* be the corresponding quotient space. X^* is homeomorphic to a familiar space. What is the space and why? HINT: Use Exercise 22.2(b).

(b) Repeat part (a) for the equivalence relation $(x_0, y_0) \sim (x_1, y_1)$ if and only if $x_0^2+y_0^2 = x_1^2+y_1^2$.

51.1. Prove that if $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic, then $k \circ h$ and $k' \circ h'$ are homotopic. HINT: Use the definition of homotopic and verify the details. You may assume compositions of continuous functions are continuous.

51.3(a). A space X is *contractible* if the identity map $i_X : X \to X$ is nulhomotopic. Prove that $I = [0, 1]$ and $\mathbb{R}$ are contractible.