# Real Analysis 1, MATH 5210

## Homework 11, The General Lebesgue Integral

Due Friday, April 29, at 3:00

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

**4.28.** Let $f$ be integrable over $E$ and $C$ a measurable subset of $E$. Prove that $\int_C f = \int_E f \cdot \chi_C$. HINT: For $h$ be a bounded measurable function of finite support $E_0$ and $0 \le h \le f^+ \cdot \chi_C$ on $E$, prove that $\int_E h = \int_C h$ and use this to show $\int_E f^+ \cdot \chi_C \le \int_C f^+$. For $h$ be a bounded measurable function of finite support $E_0$ and $0 \le h \le f^+$ on $C$, extend $h$ from $C$ to a function $h'$ on $E$ and use this extension to show $\int_E f^+ \cdot \chi_c \ge \int_C f^+$.

**4.29.** For a measurable function $f$ on $[1, \infty)$ which is bounded on bounded sets, define $a_n = \int_{[n,n+1)} f$ for each $n \in \mathbb{N}$.

**(a)** If it true that $f$ is integrable over $[1, \infty)$ if and only if the series $\sum_{n=1}^{\infty} a_n$ converges? NOTE: If the result is true, prove the two implications. If it is false in one implication, give an example illustrating the false implication and give a proof for the true implication. If both implications are false, give two examples illustrating the falseness.

**(b)** If it true that $f$ is integrable over $[1, \infty)$ if and only if the series $\sum_{n=1}^{\infty} a_n$ converges absolutely? NOTE: If the result is true, prove the two implications. If it is false in one implication, give an example illustrating the false implication and give a proof for the true implication. If both implications are false, give two examples illustrating the falseness.

NAME ______________________________ STUDENT NUMBER _______________________

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. **Do your own work!!!** Due Tuesday, March 3 at 10:30.

**4.31. Take Home Final Test Question #5.** Let $f$ be a measurable function on $E$ which can be expressed as $f = g + h$ on $E$, where $g$ is finite and integrable over $E$ and $h$ is nonnegative on $E$. Define $\int_E f = \int_E g + \int_E h$. Prove that this is properly defined in the sense that it is independent of the particular choice of finite integrable function $g$ and nonnegative function $h$ whose sum is $f$. HINT: Suppose $f = g_1 + h_1 = g_2 + h_2$ where the functions are as described. Consider cases depending on the integrability of $h_1$ and $h_2$. It appears that Royden and Fitzpatrick is missing a definition needed to complete the proof. You may assume for measurable, integrable $g$ on $E$ and measurable, nonnegative $h$ for which $\int_E h = \infty$ that $\int_E g + h = \infty$.