Real Analysis 1, MATH 5210, Spring 2025

Homework 10 (optional), Section 8.2. Weak Sequential Convergence in L^p

Due Saturday, May 3, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

These are optional questions, so clearly indicate which one(s) you want to included as part of your grade.

8.12. Prove that the sequence of Radamacher functions $\{f_n\}$ does not converge to $f \equiv 0$ in $L^p([0,1])$ where $1 \leq p \leq \infty$.

8.13. **(b)** Fix $\alpha, \beta \in \mathbb{R}$. For each $n \in \mathbb{N}$, consider the step function f_n defined on $I = [0,1]$ by

$$f_n(x) = \frac{(1-(-1)^k)\alpha}{2} + \frac{(1+(-1)^k)\beta}{2} \text{ for } \frac{k}{2^n} \leq x < \frac{(k+1)}{2^n} \text{ and } 0 \leq k < 2^n - 1$$

(also define $f(1) = \alpha$). For $\alpha \neq \beta$, prove that no subsequence of $\{f_n\}$ converges strongly in $L^p(I)$.