Real Analysis 1, MATH 5210, Spring 2025

Homework 3, Section 7.2. The Inequalities of Young, Holder, and Minkowski

Due Saturday, February 8, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

7.7(a). Let $E = (0, 1]$ and let $1 \leq p_1 < p_2 \leq \infty$. Show that $f(x) = x^\alpha$, where $-1/p_1 < \alpha < -1/p_2$, satisfies $f \in L^{p_1}(E)$ but $f \notin L^{p_2}(E)$.

7.12. For $1 \leq p < \infty$ and a sequence $a = (a_1, a_2, \ldots) \in \ell^p$, define T_a to be the function on the interval $[1, \infty)$ that takes the value a_k on $[k, k + 1)$, for $k = 1, 2, \ldots$. In parts (a) and (b) we give a proof of Hölder's Inequality for ℓ^p using Hölder's Inequality in $L^p[1, \infty)$.

(a) Show that $T_a \in L^p[1, \infty)$ and $\|a\|_p = \|T_a\|_p$.

(b) Prove a Hölder's Inequality for ℓ^p.