Real Analysis 1, MATH 5210, Spring 2025

Homework 4, Section 7.2. The Inequalities of Young, Holder, and Minkowski, Section 7.3. L^p is Complete: The Riesz-Fischer Theorem

Due Saturday, February 15, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu).

7.12. For $1 \leq p < \infty$ and a sequence $a = (a_1, a_2, \ldots) \in \ell^p$, define T_a to be the function on the interval $[1, \infty)$ that takes the value a_k on $[k, k+1)$, for $k = 1, 2, \ldots$. In parts (c) and (d) we give a proof of Minkowski's Inequality for ℓ^p using Minkowski's Inequality in $L^p[1, \infty)$.

(c) For $a \in \ell^p$, define a^*, show $a^* \in \ell^q$, and that $\displaystyle\sum_{k=1}^{\infty} a_k a_k^* = \|a\|_p$.

(d) Prove a Minkowski Inequality for ℓ^p. NOTE: The Minkowski Inequality allows us to conclude that ℓ^p is a normed linear space.

7.26. The L^p Lebesgue Dominated Convergence Theorem.
Let $\{f_n\}$ be a sequence of measurable functions that converge pointwise a.e. on E to f. For $1 \leq p < \infty$, suppose there is a function $g \in L^p(E)$ such that for all $n \in \mathbb{N}$, $|f_n| \leq g$ a.e. on E. Then $\{f_n\} \to f$ in $L^p(E)$.