Real Analysis 1, MATH 5210, Spring 2025

Homework 5, Section 7.3. L^p is Complete: The Riesz-Fischer Theorem, Section 7.4. Approximation and Separability

Due Saturday, February 22, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

7.34. **(b)** Prove that the ℓ^p spaces are Banach spaces for $1 \leq p \leq \infty$. This is the Riesz-Fischer Theorem for ℓ^p. HINT. You may assume 7.34(a): Let $1 \leq p \leq \infty$. Prove that every rapidly Cauchy sequence in ℓ^p converges with respect to the ℓ^p norm to an element of ℓ^p.

7.36. Let $\mathcal{S}$ be a subset of a normed linear space X. Prove that $\mathcal{S}$ is dense in X if and only if each $g \in X$ is the limit of a sequence in $\mathcal{S}$.

7.37. Let X be a normed linear space with $\mathcal{F} \subset \mathcal{G} \subset \mathcal{H} \subset X$. Prove that if $\mathcal{F}$ is dense in $\mathcal{G}$ and if $\mathcal{G}$ is dense in $\mathcal{H}$, then $\mathcal{F}$ is dense in $\mathcal{H}$.