

Real Analysis 1, MATH 5210, Spring 2025

Homework 7, Hong/Wang/Gardner Section 5.2. Inner Product Spaces

Due Saturday, March 29, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu).

5.2.3. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in an inner product space. Prove that $\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2)$. This is called the *Parallelogram Law*. We can interpret this geometrically in $\mathbb{R}^n$ in the sense that $\mathbf{u}$ and $\mathbf{v}$ determine a parallelogram with diagonals of length $\|\mathbf{u} + \mathbf{v}\|$ and $\|\mathbf{u} - \mathbf{v}\|$.

5.2.6. Prove that the inner product is continuous. That is, if $(\mathbf{u}_n) \rightarrow \mathbf{u}$ and $(\mathbf{v}_n) \rightarrow \mathbf{v}$, then $\langle \mathbf{u}_n, \mathbf{v}_n \rangle \rightarrow \langle \mathbf{u}, \mathbf{v} \rangle$.