

Real Analysis 1, MATH 5210, Spring 2025

Homework 9, Section 8.1. The Riesz Representation for the

Dual of L^p , $1 \leq p < \infty$

Due Saturday, April 19, at 11:59 p.m.

Write in complete sentences and paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu).

8.1. For T a bounded linear functional on linear space X , use the definition of $\|T\|_*$ to prove that

$$\|T\|_* = \sup\{|T(f)| \mid f \in X, \|f\| = 1\}.$$

8.3. (a) Let T be a linear functional on a normed linear space X . Suppose that for all sequences $\{f_n\}_{n=1}^\infty$ in X with $\{f_n\} \rightarrow f$ where $f \in X$ we have $\{T(f_n)\} \rightarrow T(f)$ in $\mathbb{R}$. Prove that T is continuous on X .