Real Analysis 1, Test 1 Study Guide

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1.4 Algebras of Sets. Algebra of sets, algebra generated by a collection of sets, σ-algebra of sets, σ-algebra generated by a collection of sets.

Axiom of Choice. Ernst Zermelo, Axiom of Choice, well orderings (binary relation, partial ordering, total ordering), Well Ordering-Principle.

Chapter 2. Real Number Systems. Completeness Axiom, Complete Ordered Field, what does an open set of real numbers "look like"?, Lindelöf Principle, Borel sets, F_σ/G_δ, $F_{\sigma\delta}/G_{\delta\sigma}$, $f : \mathbb{R} \to \mathbb{R}$ has an F_σ set of discontinuities, how man Borel sets are there?

3.1 Introduction to Lebesgue Measure. The four desired properties of a measure, monotonicity, translation invariance, countable subadditivity.

3.2 Outer Measure. Outer measure, outer measure of an interval (Theorem 3.1), outer measure is countably subadditive (Theorem 2), countable sets are measure zero (Corollary 3), approximation of a set with a G_δ set (Theorem 5), outer measure is translation invariant (Exercise 3.7).

Measurable Sets and Lebesgue Measure. (Lebesgue) measurable, outer measure zero implies measurable (Lemma 6), the measurable sets form an algebra of sets (Lemma 7 and Corollary 8), the measurable sets form a σ-algebra, (a, ∞) is measurable (Lemma 11), the Borel sets are measurable (Theorem 12), definition of Lebesgue measure, Lebesgue measure is countably additive (Theorem 13), approximation of measurable sets with open/closed/F_σ/G_δ sets (Theorem 15), Cantor set (it's measure, cardinality and use to determine how many measurable sets).

3.4 A Nonmeasurable Set. Construction of the nonmeasurable set P and the idea behind how it is shown to be nonmeasurable.

The Banach-Tarski Paradox. Georg Cantor, shifting to infinity, the Hilbert Hotel, Hausdorff Paradox.