# Real Analysis 1, "Test 2" Study Guide

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**2.5 Countable Additivity, Continuity, and the Borel-Cantelli Lemma.** Lebesgue measure, Continuity of Measure (Theorem 2.15), almost everywhere.

**2.6 Nonmeasurable Sets.** "circle plus," translation invariance under circle plus, rationally equivalent, Axiom of Choice and a choice function, construction of set $P$, why set $P$ is not measurable.

**3.1 Sums, Products, and Compositions.** Definition of measurable function and things equivalent to it, continuous functions are measurable (Proposition 3.3), monotone functions are measurable (Proposition 3.4), linear combinations of measurable sets are measurable (Proposition 3.6).

**3.2 Sequential Pointwise Limits and Simple Approximation.** Pointwise convergence, a.e. convergence, uniform convergence, a.e. pointwise limits of measurable functions are measurable (Proposition 3.9), simple function, canonical representation of simple function, Simple Approximation Lemma, Simple Approximation Theorem.

**4.2 The Lebesgue Integral of a Bounded Measurable Function Over a Set of Finite Measure.** Lebesgue integral of simple function, integrals of simple functions which are not represented canonically (Lemma 4.1), linearity and monotonicity of simple functions, upper and lower Lebesgue integral, Lebesgue integrable, Riemann integrals and Lebesgue integrals (Theorem 4.3), all bounded measurable functions on sets of finite measure are integrable (Theorem 4.4), linearity and monotonicity of integrals of bounded measurable functions on sets of finite measure (Theorem 4.5), additivity and absolute value (Corollaries 4.6 and 4.7), convergence theorems in general, Bounded Convergence Theorem.