Chapter 6. Differentiation and Integration

Note. In this chapter we prove our (Lebesgue) version of the Fundamental Theorem of Calculus. Just as in Calculus 1 (MATH 1910), there are two parts to our fundamental theorem. One part gives the value of an integral over $[a, b]$ in terms of antiderivatives (Theorem 6.10) and the other states that the derivative of an integral is the integrand (Theorem 6.14). Along with the way, we define several new ideas (in particular, absolute continuity).

Section 6.1. Continuity of Monotone Functions

Note. In this section, we prove two properties of monotone functions. However, our big result on monotone functions appears in the next section (Lebesgue's Theorem on page 112).

Note. The following result is standard in senior-level analysis. See Theorem 4-14 of my notes for Analysis 1 (MATH 4217/5217):

`http://faculty.etsu.edu/gardnerr/4217/notes/4-2.pdf`

Theorem 6.1. The f be a monotone function on the open interval (a, b). Then f is continuous except possibly at a countable number of points in (a, b).

Note. The following is, in a sense, a converse of Theorem 6.1.

Proposition 6.2. Let C be a countable subset of the open interval (a, b). Then there is an increasing function on (a, b) that is continuous only at the points in $(a, b) \setminus C$.

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