Real Analysis; Philosophy, Background, Motivation, and Chapter 1 Study Guide

Philosophy, Background, Riemannian Motivation, and Section 1.4

The following is a *brief* list of topics covered in Chapter 1 of Royden and Fitzpatrick's *Real Analysis*, 4th edition, and the associated supplements. This list is not meant to be comprehensive, but only gives a list of several important topics.

Supplement. Introduction to Math Philosophy and Meaning.

Formalism, David Hilbert, the state of mathematics in the year 1900, Frege/ Russell/Principia, Frege's work (logic), Russell and Whitehead's work (set theory), Russell's paradox, Kurt Gödel, consistence, completeness, well-formed formulas, Gödel's Incompleteness Theorems, undecidability, the Continuum Hypothesis, Alan Sokal, mis-applications of Gödel's work (and Einstein's and quantum theory), Peano's Axioms of Arithmetic, the Principle of Mathematical Induction, the Twin Prime Conjecture, the Goldbach Conjecture.

Supplement. Essential Background for Real Analysis I (MATH 5210)

De Morgan's Laws, definition of field, definition of ordered field, definition of "$<$" and "$>$," interval, upper bound and lower bound of a set, bounded set, least upper bound, greatest lower bound, completeness, definition of the real numbers $\mathbb{R}$, definition of exponentiation with irrational exponents, Epsilon Property of Sup and Inf, Cauchy sequence, subsequence, subsequential limit, $\limsup a_n$ and $\liminf a_n$, definition and properties of open/closed sets of real numbers, Classification of Open Sets of Real Numbers (Theorem 0.7), open cover, Heine-Borel Theorem, separation of a set, connected set, same cardinality, finite set, infinite set, countable set, union of a countable collection of countable sets is countable (Theorem 0.10), $\mathbb{R}$ is uncountable (Theorem 0.11 and Note following it), cardinal numbers and their ordering, Cantor's Theorem (Theorem 0.12), the Continuum Hypothesis, Alephs.

Supplement. The Riemann-Lebesgue Theorem.

Partition, upper Riemann sum, lower Riemann sum, upper Riemann integral, lower Riemann integral, Riemann integrable, Riemann integral, Riemann Condition for Integrability (Theorem 6-4), norm, ε-δ condition for Riemann integrability (Theorem 6-6), continuous functions are Riemann integrable (Theorem 6-7), measure zero set, the union of a countable collection of sets of measure zero is of measure zero (Theorem 6-9), a countable set has measure zero (Corollary 6-9), oscillation, oscillation/continuity (Theorem 6-10), the Riemann-Leabesgue Theorem (Parts a and b), for $f : [a, b] \to \mathbb{R}$ the set of discontinuities is F_σ, Dirichlet function, uniform convergence, the convergence theorem for Riemann integrals (Theorem 8.3), the two main reasons for going beyond Riemann integration.

1.4. Borel Sets.

Open and closed sets, an algebra of sets, algebra generated by a set, σ-algebra, σ-algebra generated be a set, Borel sets, cardinality of the set of Borel sets, F_σ and G_δ sets, Young's Theorem (Problem 1.56), converse of Young's Theorem, $F_{\sigma\delta}$ and $G_{\delta\sigma}$ etc. sets, continuous functions defined on $\mathbb{R}$ converge on an $F_{\sigma\delta}$ set (Problem 1.57).