

# Real Analysis, Chapter 3 Study Guide

## Chapter 3. Lebesgue Measurable Functions

The following is a *brief* list of topics covered in Chapter 3 of Royden and Fitzpatrick's *Real Analysis*, 4th edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

### 3.1. Sums, Products, and Compositions.

Equivalent properties of measurable functions (Proposition 3.1), inverse images of open sets under measurable functions (Proposition 3.2), monotone functions on intervals are measurable (Proposition 3.4), restrictions of measurable functions are measurable (Proposition 3.5(ii)), linear combinations of measurable functions are measurable (Proposition 3.6), the composition of a measurable function and a continuous function is measurable (Proposition 3.7), pointwise max and min of a finite collection of measurable functions is measurable (Proposition 3.8)

### 3.2. Sequential Pointwise Limits and Simple Approximation.

Pointwise a.e. limits of a sequence of measurable functions is measurable (Proposition 3.9), characteristic function, simple function and canonical representation, the Simple Approximation Lemma, the Simple Approximation Theorem.

### 3.3. Littlewood's Three Principals, Egoroff's Theorem, and Lusin's Theorem.

The *idea* of "nearly," Littlewood's First Principle (Theorem 2.12), Littlewood's Second Principle (Lusin's Principle), Littlewood's Third Principle (Egoroff's Theorem).