Real Analysis, Chapter 4 Study Guide

Chapter 4. Lebesgue Integration

The following is a *brief* list of topics covered in Chapter 4 of Royden and Fitzpatrick's *Real Analysis*, 4th edition.This list is not meant to be comprehensive, but only gives a list of several important topics.

4.2. Lebesgue Integral of a Bounded Measurable Function over a Set of Finite Measure.

Integral of a simple function, integral of a simple function in terms of a non-canonical representation (Lemma 4.1), linearity and monotonicity for simple functions (Proposition 4.2), upper and lower Lebesgue integrals, Lebesgue integral, Riemann and Lebesgue agree (Theorem 4.3), bounded measurable functions on sets of finite measure are Lebesgue integrable (Theorem 4.4), linearity and monotonicity for bounded measurable functions on sets of finite measure, additivity for bounded measurable functions on sets of finite measure (Corollary 4.6), the uniform limit of a sequence of bounded measurable functions on sets of finite measure is the integral of the limit of the sequence (Proposition 4.8), Bounded Convergence Theorem, an example that the integral of a sequence may not be the limit of the sequence of integrals.

4.3. Lebesgue Integral of a Measurable Nonnegative Function.

Definition of the integral of a nonnegative measurable function, Chebychev's Inequality, integrals are 0 if and only if $f = 0$ a.e. (Proposition 4.9), linearity and monotonicity for nonnegative measurable functions (Theorem 4.10), additivity (Theorem 4.11), Fatou's Lemma, examples showing that Fatou's Lemma may involve a strict inequality, Monotone Convergence Theorem, application of MCT to series (Corollary 4.12), *integrable*, Beppo Levi's Lemma.

4.4. General Lebesgue Integral.

Positive and negative parts of a function, integrable measurable function, Integral Comparison Test (Proposition 4.16), linearity and monotonicity (Theorem 4.17), additivity (Corollary 4.18), Lebesgue Dominated Convergence Theorem, General Lebesgue Dominated Convergence Theorem.

4.5. Countable Additivity and Continuity of Integration.

Countable additivity of integration (Theorem 4.20), continuity of integration (Theorem 4.21).

4.6. Uniform Integrability: The Vitali Convergence Theorem.

ε/δ behavior of integrable functions (Proposition 4.23), uniformly integrable family, pointwise a.e. limits of uniformly integrable sequence is integrable (Proposition 4.25), Vitali Convergence Theorem, pointwise a.e. convergence to 0 yields a convergence theorem if and only if the sequence is uniformly integrable (Theorem 4.26).