# Real Analysis, Chapter 5 Study Guide

## Chapter 5. Lebesgue Integration: Further Topics

The following is a *brief* list of topics covered in Chapter 5 of Royden and Fitzpatrick's *Real Analysis*, 4th edition.This list is not meant to be comprehensive, but only gives a list of several important topics.

**5.1. Uniform Integrability and Tightness: A General Vitali Convergence Theorem.**

The integral of an integrable function can be made small (Proposition 5.1), a tight family off measurable functions, the General Vitali Convergence Theorem, pointwise a.e. convergence to 0 yields a convergence theorem if and only if the sequence is uniformly integrable and tight (Corollary 5.2).

**5.2. Convergence in Measure.**

Convergence in measure, finiteness and pointwise convergence implies convergence in measure (Proposition 5.3), example of convergence in measure that is nowhere pointwise convergent, convergence in measure implies subsequence which converges pointwise a.e. (Theorem 5.4), convergence theorem if and only if convergence in measure with uniform integrability and tightness (Corollary 5.5); pointwise convergence can be replaced with convergence in measure in Fatou's Lemma, MCT, LDCT, and the Vitali Convergence Theorem (Problem 5.8).

**5.3. Characterization of Riemann and Lebesgue Integrability.**

A bounded function on a set of finite measure is Lebesgue integrable if and only if the function is measurable (Theorem 5.7), Riemann and Lebesgue agree (Theorem 5.8), the history of Riemann's approach to Theorem 5.8 (last page of notes for this section).