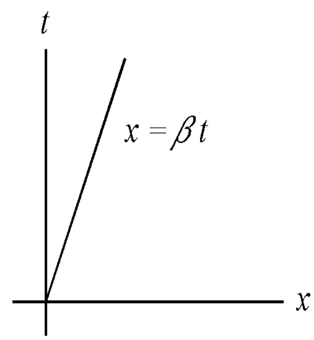
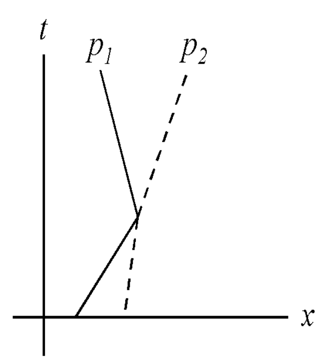
2.8 Spacetime Diagrams

Note. We cannot (as creatures stuck in 3 physical dimensions) draw the full 4 dimensions of spacetime. However, for rectilinear or planar motion, we can depict a particle's movement. We do so with a spacetime diagram in which spatial axes (one or two) are drawn as horizontal axes and time is represented by a vertical axis. In the xt−plane, a particle with velocity β is a line of the form $x = \beta t$ (a line of slope $1/\beta$):



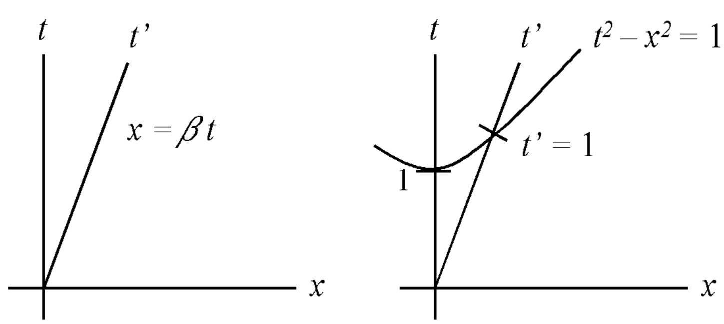
Two particles with the same spacetime coordinates must be in collision:



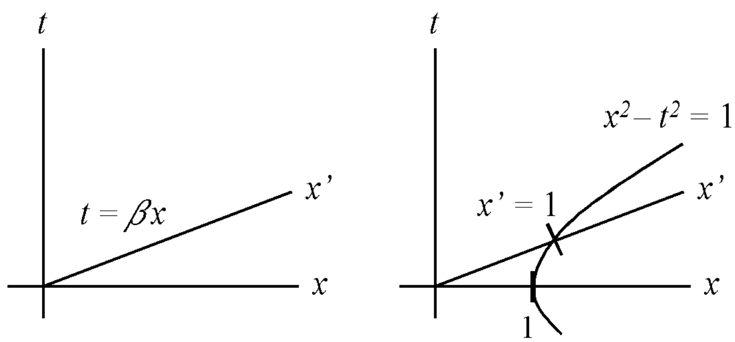
Note. The picture on the cover of the text is the graph of the orbit of the Earth as it goes around the Sun as plotted in a 3-D spacetime.

Definition. The curve in 4-dimensional spacetime which represents the relationships between the spatial and temporal locations of a particle is the particle's *world-line.*

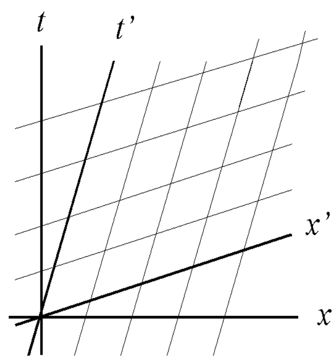
Note. Now let's represent two inertial frames of reference S and S' (considering only the $xt-$plane and the $x't'-$plane). Draw the x and t axes as perpendicular (as above). If the systems are such that $x = 0$ and $x' = 0$ coincide at $t = t' = 0$, then the point $x' = 0$ traces out the path $x = \beta t$ in S. We define this as the t' axis:



The hyperbola $t^2 - x^2 = 1$ in S is the same as the "hyperbola" $t'^2 - x'^2 = 1$ in S' (invariance of the interval). So the intersection of this hyperbola and the t' axis marks one time unit on t'. Now from equation (90b) (with $t' = 0$) we get $t = \beta x$ and define this as the x' axis. Again we calibrate this axis with a hyperbola ($x^2 - t^2 = 1$):



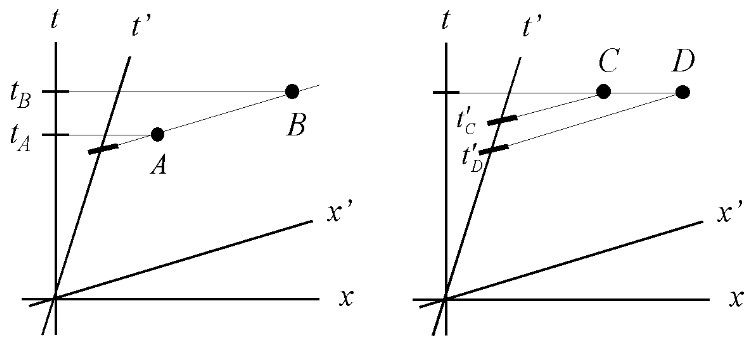
We therefore have:



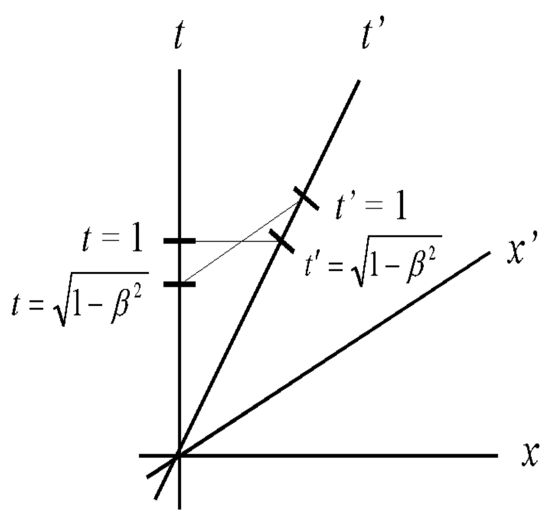
and so the S' coordinate system is oblique in the S spacetime diagram.

Note. In the above representation, notice that the larger β is, the more narrow the "first quadrant" of the S' system is and the longer the x' and t' units are (as viewed from S).

Note. Suppose events A and B are simultaneous in S' They need not be simultaneous in S. Events C and D simultaneous in S need not be simultaneous in S'.

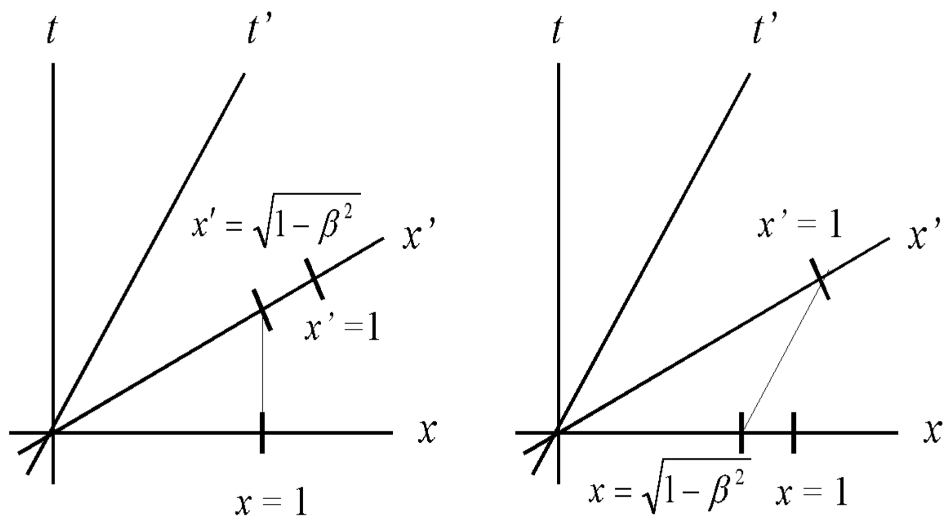


Note. A unit of time in S is dilated in S' and a unit of time in S' is dilated in S.



Note. Suppose a unit length rod lies along the x axis. If its length is measured in S' (the ends have to be measured simultaneously in S')

then the rod is shorter. Conversely for rods lying along the x' axis.



Example (Exercise 2.8.4). An athlete carrying a pole 16m long runs toward the front door of a barn so rapidly that an observer in the barn measures the pole's length as only 8m, which is exactly the length of the barn. Therefore at some instant the pole will be observed entirely contained within the barn. Suppose that the barn observer closes the front and back doors of the barn at the instant he observes the pole entirely contained by the barn. What will the athlete observe?

Solution. We have two events of interest:

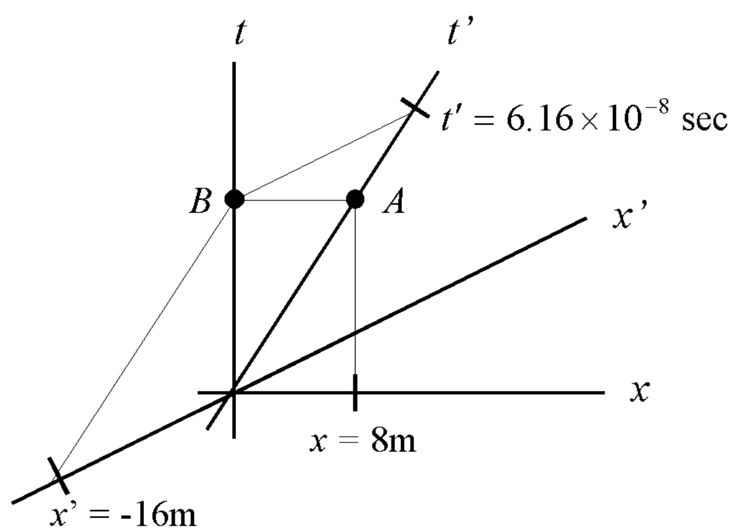
A = The front of the pole is at the back of the barn.

B = The back of the pole is at the front of the barn.

From Exercise 2.8.3, the observer in the barn (frame S) observes these events as simultaneous (each occuring at $t_{AB} = 3.08 \times 10^{-8}$sec after the front of the pole was at the front of the barn). However, the athlete observes event A after he has moved the pole only 4m into the barn. So for him, event A occurs when $t'_A = 1.54 \times 10^{-8}$sec. Event B does not occur until the pole has moved 16m (from $t' = 0$) and so event B occurs for the athlete when $t'_B = 6.16 \times 10^{-8}$sec. Therefore, the barn observer observes the pole totally within the barn (events A and B), slams the barn doors, and observes the pole start to break through the back of the barn all simultaneously. The athlete first observes event A along with the slamming of the back barn door and the pole starting to break through this door (when $t' = 1.54 \times 10^{-8}$sec) and THEN observes the front barn door slam at $t'_B = 6.16 \times 10^{-8}$sec. The spacetime diagram is:

A occurs at
$t' = 1.54 \times 10^{-8}$ sec

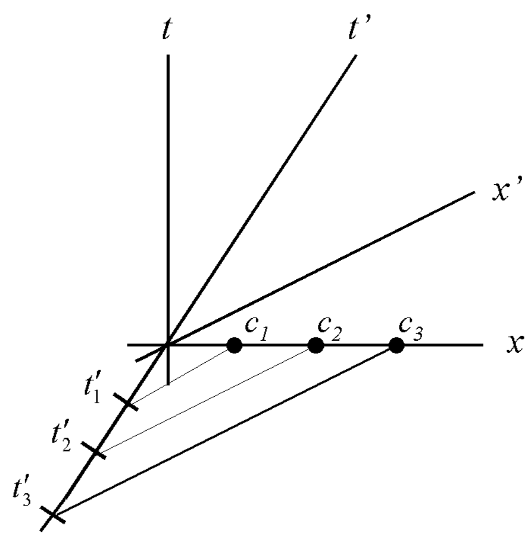
B occurs at
$t = 3.08 \times 10^{-8}$ sec



Since the order of events depends on the frame of reference, the apparent paradox is explained.

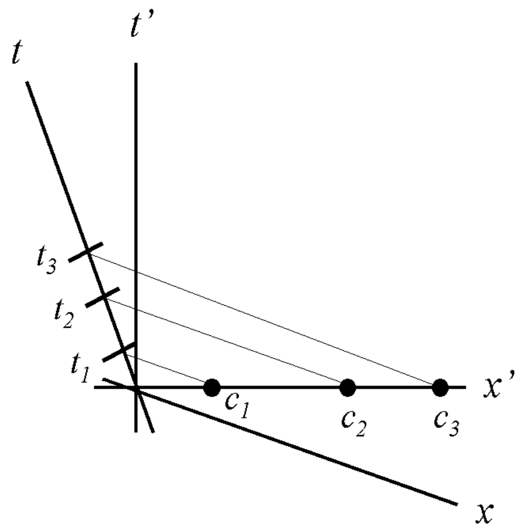
Example (Exercise 2.8.5). Using a diagram similar to Figure II-15, show that (a) at time $t = 0$ in the laboratory frame, the rocket clocks that lie along the positive $x-$axis are observed by S to be set behind the laboratory clocks, with the clocks further from the origin set further behind, and that (b) at time $t' = 0$ in the rocket frame, the laboratory clocks that lie along the positive $x'-$axis are observed by S' to be set further ahead.

Solution. (a) At $t = 0$, clocks in S' that lie along the positive $x-$axis are observed by S to be behind the S clocks, with the clocks further from the origin set further behind:



So clocks c_i arranged as in the figure read times $0 > t'_1 > t'_2 > t'_3 > \cdots$.

(b) Conversely, in the S' frame:



So clocks c_i arranged as in the figure read times $0 < t_1 < t_2 < t_3 < \cdots$.