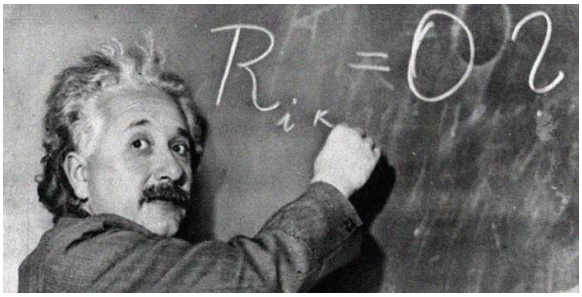
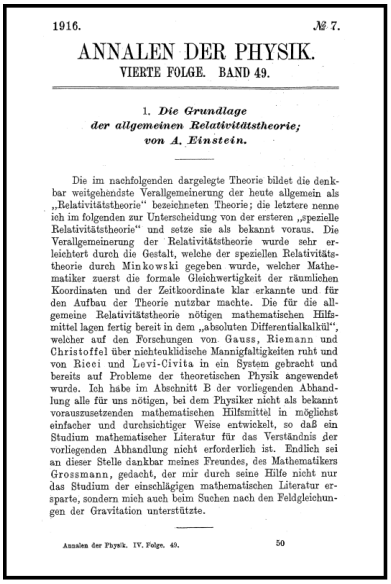
Chapter 3. General Relativity: The Geometry of Curved Spacetime

Note. As we have seen, the Special Theory of Relativity deals only with inertial (unaccelerated) observers. Such observers cannot be under the influence of a gravitational field, and one might say that special relativity describes the mechanics in a massless universe!





1916. № 7.

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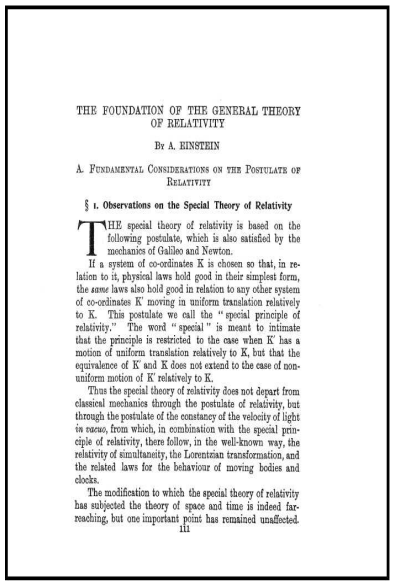
VIERTE FOLGE. BAND 49.

1. ***Die Grundlage der allgemeinen Relativitätstheorie;***
von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathematiker zuerst die formale Gleichwertigkeit der räumlichen Koordinaten und der Zeitkoordinate klar erkannte und für den Aufbau der Theorie nutzbar machte. Die für die allgemeine Relativitätstheorie nötigen mathematischen Hilfsmittel lagen fertig bereit in dem „absoluten Differentialkalkül", welcher auf den Forschungen von Gauss, Riemann und Christoffel über nichteuklidische Mannigfaltigkeiten ruht und von Ricci und Levi-Civita in ein System gebracht und bereits auf Probleme der theoretischen Physik angewendet wurde. Ich habe im Abschnitt B der vorliegenden Abhandlung alle für uns nötigen, bei dem Physiker nicht als bekannt vorauszusetzenden mathematischen Hilfsmittel in möglichst einfacher und durchsichtiger Weise entwickelt, so daß ein Studium mathematischer Literatur für das Verständnis der vorliegenden Abhandlung nicht erforderlich ist. Endlich sei an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann, gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichungen der Gravitation unterstützte.

Annalen der Physik. IV. Folge. 49. 50



THE FOUNDATION OF THE GENERAL THEORY OF RELATIVITY

By A. EINSTEIN

A. Fundamental Considerations on the Postulate of Relativity

§ 1. Observations on the Special Theory of Relativity

THE special theory of relativity is based on the following postulate, which is also satisfied by the mechanics of Galileo and Newton.

If a system of co-ordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form, the *same* laws also hold good in relation to any other system of co-ordinates K' moving in uniform translation relatively to K. This postulate we call the "special principle of relativity." The word "special" is meant to intimate that the principle is restricted to the case when K' has a motion of uniform translation relatively to K, but that the equivalence of K' and K does not extend to the case of non-uniform motion of K' relatively to K.

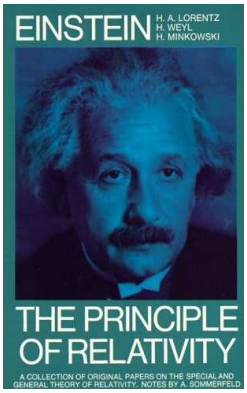
Thus the special theory of relativity does not depart from classical mechanics through the postulate of relativity, but through the postulate of the constancy of the velocity of light *in vacuo*, from which, in combination with the special principle of relativity, there follow, in the well-known way, the relativity of simultaneity, the Lorentzian transformation, and the related laws for the behaviour of moving bodies and clocks.

The modification to which the special theory of relativity has subjected the theory of space and time is indeed far-reaching, but one important point has remained unaffected.

111

Einstein with his field equations, the original 1916 paper in German, and an English translation. Images are from `www.amnh.org/exhibitions/einstein/`, `hermes.ffn.ub.es/luisnavarro/nuevo_maletin/Einstein_GRelativity_1916.pdf`, and `myweb.rz.uni-augsburg.de/~eckern/adp/history/einstein-papers/1916_49_769-822.pdf`, respectively. (Accessed July 3, 2016.)

Note. In order to deal with accelerating frames of reference or frames of reference under the influence of gravity, the Special Theory of relativity has to be extended. This was accomplished by Einstein in his "Die Grundlage der allgemeinen Relativitätstheorie" ("The Foundation of the General Theory of Relativity") in *Annelan der Physik* (*Annals of Physics*) **49**(7), 1916. As we will see, he considered gravity not as a force, but as a curvature of spacetime. Falling objects, planets in orbits, and rays of light then are observed to follow geodesics in curved spacetime. (Surprisingly, the picture on the cover of the text is a "straight" geodesic in a curved spacetime!) An English translation of Einstein's paper appears in *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity* by H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, Dover Publications (1952). This is a republication of the 1923 publication by Methuen and Company. It also includes Einstein's 1905 paper on special relativity, Einstein's 1905 paper on "$E = mc^2$, and Minkowski's 1908 address to the 80th Assembly of German natural Scientists and Physicians, "Space and Time,"in which he defines the currently-named "Minkowski spacetime." Most of the work can likely be found online (the web link to the 1916 paper given above is to a reprint from the Dover book).



The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity by H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, Dover Publications (1952).

Note. After some introductory material, we will discuss geodesics in the semi-Riemannian 4-manifold of spacetime (Section 6) and "outline" the reasoning which lead Einstein to his field equations (Section 7). We will then solve the field equations outside an isolated sphere of mass M. Finally, we'll explore orbits and the "bending of light" under the General Theory of Relativity.

Revised: 6/30/2019