Differential Geometry (and Relativity) - Summer 2019

Homework 2, Sections 1.2 and 1.3

Due Tuesday, June 11 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why.

I.2.3. Find the Gauss curvature of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at the end points of its three axes, i.e., at the points $\pm(a, 0, 0)$, $\pm(0, b, 0)$, and $\pm(0, 0, c)$. You may use the results of Exercise I.1.7.

I.3.2(d) For the surface of revolution

$$\vec{X}(u, v) = (a \cosh u \cos v, a \cosh u \sin v, b \sinh u),$$

sketch the profile curve ($v = 0$) in the xz-plane, and then sketch the surface. In each case, prove that $\vec{X}$ is regular and give an equation of the surface in the form $g(x, y, z) = 0$. Assume a and b are positive constants.

I.3.7(a,b) Let $\vec{\alpha}(u) = (\cos u, \sin u, 0)$. Through each point of $\vec{\alpha}(u)$, pass a unit line segment with midpoint $\vec{\alpha}(u)$ and direction vector

$$\vec{\beta}(u) = \left(\sin \frac{u}{2}\right) \vec{\alpha}(u) + \left(\cos \frac{u}{2}\right) (0, 0, 1).$$

The resulting surface,

$$\vec{X}(u, v) = \vec{\alpha}(u) + v\vec{\beta}(u), \; -\frac{1}{2} \leq v \leq \frac{1}{2}$$

is called a *Möbius strip*. (a) Write out the coordinate functions of $\vec{X}(u, v)$. (b) Sketch the rulings for $u = 0$, $\pi/2$, π, and $3\pi/2$.

I.3.7(c) Connect the end points of these rulings by drawing the u-parameter curve $\vec{X}(u, 1/2)$ for $0 \leq u \leq 4\pi$. Note that the u parameter curves are closed and have period 4π, except for $\vec{\alpha}(u) = \vec{X}(u, 0)$, which has period 2π.