Differential Geometry (and Relativity) - Summer 2019

Homework 3, Sections I.4 and I.5

Due Friday, June 14 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why.

I.4.3(a) For the surface $\vec{X}(u,v) = (R\cos u\cos v, R\sin u\cos v, R\sin v)$, compute the matrix (g_{ij}), its determinant g, the inverse matrix (g^{ij}), and the unit normal vector $\vec{U}$.

I.4.5. Compute the metric form and the unit normal vector $\vec{U}$ for the general surface of revolution $\vec{X}(u,v) = (f(u)\cos v, f(u)\sin v, g(u))$.

I.4.13. The right circular cylinder $x^2 + y^2 = R^2$ (in E^3) may be parametrized as

$$\vec{X}(u,v) = \left(R\cos\frac{u}{R}, R\sin\frac{u}{R}, v\right).$$

Compute the metric form. (If we endow the uv-pane with the Euclidean metric $ds^2 = du^2 + dv^2$, then the result of this exercise shows that any curve in the uv-plane and its image under $\vec{X}$ on the cylinder have the same length. A smooth mapping such as $\vec{X}$, which preserves lengths of curves is called a *local isometry*. An *isometry* is a local isometry that is one to one and onto.)

I.5.3(a) Compute the second fundamental form of the helicoid

$$\vec{X}(u,v) = (u\cos v, u\sin v, bv) \text{ where } b \text{ is a constant.}$$

I.5.8. (Bonus.) Let

$$\vec{X}(u^1,u^2) = \left(x(u^1,u^2), y(u^1,u^2), z(u^1,u^2)\right)$$

be a surface. Prove that

$$L_{ij} = \frac{\det\begin{bmatrix} x_{ij} & y_{ij} & z_{ij} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{bmatrix}}{\sqrt{g}}.$$