Differential Geometry (and Relativity) - Summer 2016

Homework 4, Sections I.6 and I.7

Due Wednesday, June 22 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why.

I.6.1(a) In Examples 14 and 16 of the class notes, we saw that the curvature of surface $z = f(x, y)$ is

$$K = \frac{f_{xx}f_{yy} - f_{xy}^2}{(1 + f_x^2 + f_y^2)^2}.$$

Use this formula to compute the Gauss curvature of the hyperbolic paraboloid $z = (y^2 - x^2)/2$. NOTE: You will notice from the solution that the hyperbolic paraboloid is of negative curvature at all points, but is not of constant curvature (and so is not an acceptable model for non-Euclidean geometry). The greater the distance from the origin, the "flatter" the surface is.

I.6.2(a) Show that the surface of revolution $\vec{X}(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$ has Gauss curvature

$$K = \frac{g' \det \begin{pmatrix} f' & f'' \\ g' & g'' \end{pmatrix}}{f((f')^2 + (g')^2)^2}.$$

HINT: Several relevant parameters are already calculated in Exercise I.5.2 in the class notes. Use the fact that $K = L/g$.

I.6.4(a). Compute the Gauss curvature of the helicoid

$$\vec{X}(u, v) = (u\cos v, u\sin v, bv) \text{ where } b \text{ is constant.}$$

HINT: You may use the results of Exercise I.5.3(a) which you have already computed.

I.7.1. Show that the geodesic curvature $k_g = \vec{U} \cdot \vec{\alpha}' \times \vec{\alpha}''$ is given by $k_g = k\vec{U} \cdot \vec{B} = k\cos\theta$ where k is the curvature of $\vec{\alpha}$ and θ is the angle between the osculating plane of $\vec{\alpha}$ and the tangent plane of the surface.