Differential Geometry (and Relativity) - Summer 2016

Homework 5, Sections I.8 and I.9

Due Friday, June 24 at 11:20

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why.

I.8.1(a). Use Equation (59),

$$K = \frac{-1}{2\sqrt{EG}} \left\{ \frac{\partial}{\partial u}\left[\frac{G_u}{\sqrt{EG}}\right] + \frac{\partial}{\partial v}\left[\frac{E_v}{\sqrt{EG}}\right] \right\},$$

to compute the Gauss curvature of the sphere

$$\vec{X}(u,v) = (R\cos u \cos v, R\sin u\cos v, R\sin v).$$

I.8.4. Suppose a surface M has a metric form $ds^2 = du^2 + e^{2u/k}dv^2$. Use equation (59) to show that the Gauss curvature is $K = -1/k^2$.

I.9.6(a). Suppose that for $M = \{(u,v) \mid u \in \mathbb{R}, v > 0\}$, we have $ds^2 = \dfrac{k^2}{v^2}(du^2 + dv^2)$ where k is a positive constant. This forms the *Poincare Upper Half-Plane*. Use Exercise I.9.4, $K = \gamma(\gamma_{uu} + \gamma_{vv}) - (\gamma_u^2 + \gamma_v^2)$, to prove that $K = -1/k^2$. Here, $\gamma(u,v) = v/k$.

I.9.6(b). By Exercise I.7.14, a geodesic of M satisfies

$$\frac{du}{dv} = \frac{hv}{(k^2 - h^2v^2)^{1/2}}$$

for some constant h. Show that the solutions to this differential equation are semicircles centered on the x axis and (when $h = 0$) rays perpendicular to the u-axis.