Differential Geometry (and Relativity) - Summer 2019

Homework 5, Sections I.7 and I.8 (Revised)

Due Monday, June 23 at 1:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why.

I.7.2(a) Use Exercise I.7.1 to find the geodesic curvature of a circle of latitude on sphere (a u-parameter curve in Example 7 in Section I.3). Such a circle can be given parametrically as

$$\vec{\alpha}(u) = (R\cos v^* \cos u, R\cos v^* \sin u, R\sin v^*)$$

where v^* is fixed.

I.7.12. Suppose surface M has metric form $ds^2 = Edu^2 + Gdv^2$, with $E_v = G_v = 0$.

(a) Verify that the only non-zero Christoffel symbols Γ^r_{ij} are

$$\Gamma^1_{11} = \frac{E_u}{2E},\ \Gamma^2_{12} = \Gamma^2_{21} = \frac{G_u}{2G},\ \Gamma^1_{22} = -\frac{G_u}{2E}.$$

(b) Show that a geodesic on M satisfies $v'' + \frac{G_u}{G}u'v' = 0$ and integrate this equation to obtain $Gv' = h$ (a non-zero constant). HINT: Produce a differential equation based on the Christoffel symbols Γ^2_{ij}, divide it by v', and solve by integration.

I.8.1(a). Use Equation (59),

$$K = \frac{-1}{2\sqrt{EG}}\left\{\frac{\partial}{\partial u}\left[\frac{G_u}{\sqrt{EG}}\right] + \frac{\partial}{\partial v}\left[\frac{E_v}{\sqrt{EG}}\right]\right\},$$

to compute the Gauss curvature of the sphere

$$\vec{X}(u, v) = (R\cos u \cos v, R\sin u \cos v, R\sin v).$$

I.8.1(c). Use Equation (59),

$$K = \frac{-1}{2\sqrt{EG}}\left\{\frac{\partial}{\partial u}\left[\frac{G_u}{\sqrt{EG}}\right] + \frac{\partial}{\partial v}\left[\frac{E_v}{\sqrt{EG}}\right]\right\},$$

to compute the Gauss curvature of the torus

$$\vec{X}(u, v) = ((R + r\cos u)\cos v, (R + r\cos u)\sin v, r\sin u)$$

where $r > 0$ and $R \geq r$.