Write in complete sentences!!! Explain what you are doing and convince me that you understand what you are doing and why. Do any four of the following.

III.8.2. Compute any three of the Christoffel symbols in Equation (153) on page 214 (and on page 3 of the notes for Section III.8).

III.8.4. Assume the interval is given by
\[ d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\phi^2 - r^2 \sin^2 \phi d\theta^2. \]

By setting \( d\tau = 0 \) and integrating, compute the time \( \int dt \) required for a light photon to travel radially (i.e., with \( \phi \) and \( \theta \) constant) from \( r = r_1 \) to \( r = r_2 \).

III.9.3. Verify the values of \( \Delta \theta_{\text{cent}} \) given in Table III-2 for Mercury and Earth. HINT: \( \Delta \theta_{\text{cent}} = \frac{6\pi M n}{a(1-e^2)} \) where \( M \) is the solar mass (in cm it’s \( 1.48 \times 10^5 \)), \( n \) is the number of orbits for the planet in 100 years, \( a \) is the semi-major axis of the orbit, and \( e \) is the eccentricity of the orbit.

III.10.2. Compute the angle of deflection \( \Delta \theta = 4M_{\text{Earth}}/R_{\text{Earth}} \) for a light ray grazing the Earth (in radians and then in seconds of arc). HINT: You can convert mass from traditional units (grams, say) to geometric units (centimeters, say) with the formula \( M_{\text{cm}} = GM_{\text{grams}}/c^2 \). It follows that the mass is 1/2 the Schwarzschild radius.

Black Hole. Use the substitution \( v = t + r + 2M \ln \left| \frac{r}{2M} - 1 \right| \) to show that the Schwarzschild metric of number III.8.4 can be written as
\[ d\tau^2 = (1 - 2M/r)dv^2 - 2dv
dr - r^2 d\phi^2 - r^2 \sin^2 \phi d\theta^2. \]

HINT: Compute \( dt^2 \) in terms of \( dv \) and \( dr \), then substitute this into the Schwarzschild solution.