Differential Geometry (and Relativity) - Summer 2016

Homework 8, Sections III.8, III.9, III.10, and Black Holes

Due Friday, July 8 at 5:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Do any **four** of the following.

III.8.2. Compute any three of the Christoffel symbols in Equation (153) on page 214 (and on page 3 of the notes for Section III.8).

III.8.4. Assume the interval is given by

$$d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2\, d\phi^2 - r^2 \sin^2\phi\, d\theta^2.$$

By setting $d\tau = 0$ and integrating, compute the time ($\int dt$) required for a light photon to travel radially (i.e., with ϕ and θ constant) from $r = r_1$ to $r = r_2$.

III.9.3. Verify the values of $\Delta\theta_{\text{cent}}$ given in Table III-2 for Mercury and Earth. HINT: $\Delta\theta_{\text{cent}} = \dfrac{6\pi M n}{a(1-e^2)}$ where M is the solar mass (in cm it's 1.48×10^5), n is the number of orbits for the planet in 100 years, a is the semi-major axis of the orbit, and e is the eccentricity of the orbit.

III.10.2. Compute the angle of deflection $\Delta\theta = 4M_{\text{Earth}}/R_{\text{Earth}}$ for a light ray grazing the Earth (in radians and then in seconds of arc). HINT: You can convert mass from traditional units (grams, say) to geometric units (centimeters, say) with the formula $M_{\text{cm}} = GM_{\text{grams}}/c^2$. It follows that the mass is 1/2 the Schwarzschild radius.

Black Hole. (Bonus) Use the substitution $v = t + r + 2M \ln\left|\dfrac{r}{2M} - 1\right|$ to show that the Schwarzschild metric of number III.8.4 can be written as

$$d\tau^2 = (1 - 2M/r)dv^2 - 2dv\, dr - r^2\, d\phi^2 - r^2 \sin^2\phi\, d\theta^2.$$

HINT: Compute dt^2 in terms of dv and dr, then substitute this into the Schwarzschild solution.