Differential Geometry; Chapter IV Study Guide

The following is a *brief* list of topics covered in Chapter IV, "Metric Vector Spaces," of Dodson and Poston's *Tensor Geometry*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

IV.1. Metrics.

Bilinear form, symmetric, anti-symmetric, non-degenerate, positive/negative definite, indefinite, metric tensor, inner product, metric vector space, inner product space, standard inner product on $\mathbb{R}^n$, Lorentz metric on $\mathbb{R}^4$, determinant metric, length of a vector with respect to a metric tensor $\mathbf{G}$, spacetime interval, timelike, spacelike, lightlike, lightcones, norm, Triangle Inequality, norm induced by an inner product, size of a vector, unit vector, Schwarz's Inequality (Lemma IV.1.07), orthogonal vectors, perp space (or orthogonal complement), mappings $\mathbf{G}_\downarrow$ and $\mathbf{G}_\uparrow$ in metric vector space $(X, \mathbf{G})$, metric tensor and metric inner product $\mathbf{G}^*$ induced by metric tensor and metric inner product $\mathbf{G}$.

IV.2. Maps.

Orthogonal projection operator and resulting components of a vector, orthogonal complements in the Lorentz space $\mathbb{H}^2$, orthogonal complement $S^\perp$, decomposition of $\mathbf{x}$ as an element of subspace S and an element of subspace $S^\perp$ (Lemma IV.2.04), direct sum of subspaces S and $S^\perp$, dimensions of S and $S^\perp$ (Corollary IV.2.05), isometry (orthogonal operator or unitary operator), adjoint of a linear operator, self adjoint linear operator and properties (Lemma IV.2.A), $\mathbf{A}$ is an orthogonal operator if and only if $\mathbf{A}^T\mathbf{A} = \mathbf{I}$ (Lemma IV.2.09), $\mathbf{A}$ is orthogonal if and only it $\mathbf{A}^T$ is orthogonal (Corollary IV.2.10).

IV.3. Coordinates.

Matrix representations of $\mathbf{G}_\downarrow$ and $\mathbf{G}_\uparrow$ with respect to bases β and β' (Note IV.3.B), matrix representation of $\mathbf{G}^*$ (Note IV.3.C), orthogonal set of vectors, orthonormal set, orthonormal basis, every metric vector space has at least one orthonormal basis (Theorem IV.2.05), signature of metric $\mathbf{G}$, Sylvester's Law of Inertia (Corollary IV.3.10), the signature of $\mathbf{G}^*$ equals the signature of $\mathbf{G}$ (Corollary IV.3.12), the matrix representation relationship $[\mathbf{A}^T]_\beta^\beta = ([\mathbf{A}]_\beta^\beta)^t$ where β is an orthonormal basis (Lemma IV.3.13; T represents that adjoint of a linear operator and t represents the transpose of a matrix), entries of the matrices representing $[\mathbf{A}^T]$ and $[\mathbf{A}]^t$ with respect to an orthonormal basis (Lemma IV.3.14) and the corresponding result for self-adjoint operators (Corollary IV.3.15), orthogonal linear operators have matrix representations with orthonormal row and columns (Lemma IV.3.16).

IV.4. Diagonalizing Symmetric Operators.

Fundamental Theorem of Real Symmetric Matrices, a maximal vector for a linear operator in an inner product space (Definition IV.4.01), operator norm, symmetric linear operator, eigenvectors of $\mathbf{A}$ for eigenvalue $+\|\mathbf{A}\|$ or $-\|\mathbf{A}\|$ (Lemma IV.4.03), operator on $\mathbf{x}^\perp$ induced by $\mathbf{A}$ (Lemma IV.4.04), symmetric linear operators yield orthonormal bases of eigenvectors (Theorem IV.4.05), comments about eigenvalues with respect to difference bases (Note IV.4.A), dimension of an eigenspace for a symmetric operator (Corollary IV.4.07), eigenvalues of a symmetric linear operator are real (Corollary IV.4.08), principal directions of a symmetric bilinear form, isotropic linear operator in an inner product space, orthonormal bases of eigenvectors for Lorentz space $\mathbb{L}^4$ (Lemma IV.4.13).