Differential Geometry; Chapter V Study Guide

The following is a *brief* list of topics covered in Chapter V, "Tensors and Multilinear Forms," of Dodson and Poston's *Tensor Geometry*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

V.1. Multilinear Forms.

Multilinear mapping, mulitlinear form, tensor product of covariant vectors from the same space $\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_n \in X^*$, tensor product of covariant vectors from different spaces $\mathbf{g}_1 \in X_1^*, \mathbf{g}_2 \in X_2^*, \ldots, \mathbf{g}_n \in X_n^*$, mapping $\bigotimes : X_1^* \times X_2^* \times \cdots \times X_n^* \to L(X_1, X_2, \ldots, X_n; \mathbb{R})$ defined as $\bigotimes((\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_n)) = \mathbf{g}_1 \otimes \mathbf{g}_2 \otimes \cdots \otimes \mathbf{g}_n$, tensor product of vector spaces $X_1^*, X_2^*, \ldots, X_n^*$, noncommutivity of tensor product of vectors, simple tensor (or pure tensor), sum of simple tensors may not be simple, $\bigotimes$ is multilinear (T i), existence of a unique mapping as given in (T ii), tensor product of vector spaces $X_1, X_2, \ldots, X_n$ (Definition V.1.04 and Lemma V.1.05), properties of the tensor product of vectors (T A) and (T S) (Lemma V.1.A), $X_1^* \otimes X_2^* \otimes \cdots \otimes X_n^* \cong (X_1 \otimes X_2 \otimes \cdots \otimes X_n)^*$ (Lemma V.1.07), $L(X_1; X_2) \cong X_1^* \otimes X_2$ (Lemma V.1.08), tensor product of linear maps $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n$, tensors of X covariant of degree h and contravariant of degree k, type of a tensor, contraction map, coordinates of tensors (such as $x^{ijk}_{\ell m}$), transformations of coordinates of tensors, "contracting over j and ℓ," raising and lowering indices for coordinates with $\mathbf{G}_\uparrow$ and $\mathbf{G}_\downarrow$.