Complex Variables, MATH 4337/5337, Fall 2020

Homework 1, Sections 1.2. Basic Algebraic Properties, 1.3. Further Properties, 1.4. Vectors and Moduli

Due Tuesday, January 28 at 12:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

1.2.3. Use the properties of complex numbers given in Theorem 1.2.1 to prove that

$$(1+z)^2 = 1 + 2z + z^2.$$

1.2.8(a). Write $(x, y) + (u, v) = (x, y)$ and prove that the complex number $0 = (0, 0)$ is unique as an additive identity. You may assume that the additive identity 0 in the real numbers is unique.

1.2.8(b). Write $(x, y)(u, v) = (x, y)$ and prove that the number $1 = (1, 0)$ is the unique multiplicative identity. You may assume that the multiplicative identity 1 in the real numbers is unique. HINT: Consider two cases. First, suppose $y \neq 0$ and second suppose $y = 0$.

1.3.5. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Prove for $z_2 \neq 0$ that

$$\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}.$$

1.3.8. (Graduate) Use mathematical induction to verify the binomial formula:

$$(z_1 + z_2)^n = \sum_{k=0}^{n} \binom{n}{k} z_1^k z_2^{n-k} \text{ for } n \in \mathbb{N}.$$

1.5.8. Let z_1 and z_2 denote complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Prove that

$$|(x_1 + iy_1)(x_2 + iy_2)| = \sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}$$

and then prove $|z_1z_2| = |z_1||z_2|$.