Complex Variables, MATH 4337/5337, Spring 2020

Homework 10, 4.43. Contours, 4.46. Examples Involving Branch Cuts; Solutions

Due Thursday, April 23 at 12:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

4.43.1. If $w(t) = u(t) + iv(t)$ is continuous on an interval $t \in [a, b]$, then show the following.

(b) $\displaystyle\int_a^b w(t)\, dt = \int_\alpha^\beta w(\varphi(\tau))\varphi'(\tau)\, d\tau$ where $t = \varphi(\tau)$ is a real-valued function with continuous derivative satisfying $\varphi'(\tau) > 0$ which maps the interval $\tau \in [\alpha, \beta]$ onto the interval $t \in [a, b]$ (so that $\varphi(\alpha) = a$ and $\varphi(\beta) = b$).

4.43.6. (Graduate) Let $y(x)$ be a real-valued function defined on the interval $x \in [0, 1]$ by means of the equation

$$y(x) = \begin{cases} x^3 \sin(\pi/x) & \text{for } x \in (0, 1] \\ 0 & \text{for } x = 0. \end{cases}$$

(a) Show that the equation $z(x) = x + iy(x)$ where $x \in [0, 1]$ represents an arc C that intersects the real axis at the points $z = 1/n$ where $n \in \mathbb{N}$ and $z = 0$, as shown in Figure 38.

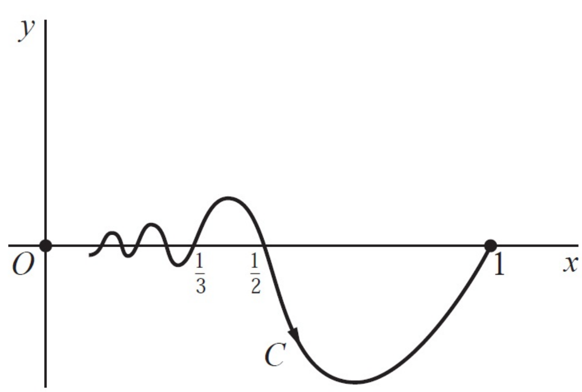

FIGURE 38

(b) Verify that the arc C is part (a) is, in fact, a *smooth* arc. First, show $z(x)$ is continuous on $[0, 1]$.

4.46.7. Use the parametric representation for C of $z(\theta) = e^{i\theta}$ where $\theta \in [0, \pi/2]$ to evaluate $\int_C f(z)\, dz$ where $f(z)$ is the principal branch of z^{-1-2i},

$$z^{-1-2i} = \exp((-1-2i)\operatorname{Log} z) \text{ where } z \neq 0 \text{ and } \operatorname{Arg}(z) \in (-\pi, \pi).$$

4.46.11. Let C denote the positively oriented right-hand half of the circle $|z| = 2$. Evaluate the integral of the function $f(z) = \overline{z}$ along C using the given parametric representation.

(b) $z(y) = \sqrt{4 - y^2} + iy$ where $y \in [-2, 2]$.

4.46.13. Let C_0 denote the circle centered at z_0 with radius R, and use the parametrization $z = z_0 + Re^{i\theta}$ where $\theta \in [-\pi, \pi]$ to show that

$$\int_{C_0} (z - z_0)^{n-1}\, dz = \begin{cases} 0 & \text{when } n \in \mathbb{Z} \setminus \{0\} \\ 2\pi i & \text{when } n = 0. \end{cases}$$