Complex Variables, MATH 4337/5337, Spring 2020

Homework 11, 4.47. Upper Bounds for Moduli of Contour Integrals, 4.53. Multiply Connected Domains, 4.57. Some Consequences of the Extension

Due Thursday, April 30 at 12:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

4.47.5. (Graduate) Let C_R be the circle $|z| = R$ (where $R > 1$) described in the counterclockwise direction. Show that

$$\left| \int_{C_R} \frac{\operatorname{Log} z}{z^2}\, dz \right| \leq 2\pi \left(\frac{\pi + \ln R}{R} \right),$$

and then use L'Hopital's Rule to show that the value of the integral tends to zero as R tends to infinity.

4.49.4. Consider the branch of $z^{1/2}$:

$$z^{1/2} = f_2(z) = \sqrt{r} e^{i\theta/2} \text{ where } \theta \in (\pi/2, 5\pi/2).$$

Let C_2 be any contour from $z = -3$ to $z = 3$ that lies in the open lower half-plane, except at its endpoints (see Figure 52 in Example 4.44.4 in the class notes; see Figure 54 in Example 4.48.4 in the 9th edition of the book). Use $f_2(z)$ to show that $\displaystyle\int_C z^{1/2}\, dz = 2\sqrt{3}(-1+i)$ where the integrand is the branch of $z^{1/2}$:

$$z^{1/2} = \exp\left(\frac{1}{2} \log z\right) = \sqrt{r} e^{i\theta/2} \text{ where } \theta \in (0, 2\pi).$$

Notice that this combines with Example 4.44.4 (Exercise 4.48.4 in the 9th edition of the book) to show $\displaystyle\int_{C_1 - C_2} z^{1/2}\, dz = -4\sqrt{3}$.

4.53.2. Let C_1 denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 1$, $y = \pm 1$ and let C_2 be the positively oriented circle $|z| = 4$ (see Figure 63; Figure 65 in the 9th edition). Use the Principle of Deformation (Corollary 4.49) to point out why $\int_{C_1} f(z)\, dz = \int_{C_2} f(z)\, dz$ for the following.

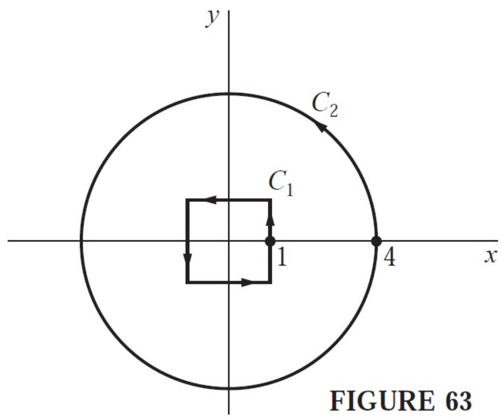

FIGURE 63

(b) $f(z) = (z+2)/\sin(z/2)$.

4.57.1. (a) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate $\displaystyle\int_C \frac{e^{-z}\,dz}{z-(\pi i/2)}$.

4.57.1. (e) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate $\displaystyle\int_C \frac{\tan(z/2)\,dz}{(z-x_0)^2}$ where $-2 < x_0 < 2$.