Complex Variables, MATH 4337/5337, Fall 2020

Homework 1, Sections 1.5. Complex Conjugates,

1.6. Exponential Form, 1.7. Products and Powers in Exponential Form, 1.8.

Arguments of Products and Quotients

Due Tuesday, February 4 at 12:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

1.6.7. Show that $|\text{Re}(2 + \overline{z} + z^3)| \leq 4$ when $|z| \leq 1$.

1.6.15. Follow the steps below to give an algebraic derivation of the Triangle Inequality $|z_1 + z_2| \leq |z_1| + |z_2|$.

(a) (Graduate) Prove that $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = z_1\overline{z}_1 + (z_1\overline{z}_2 + \overline{z_1\overline{z}_2}) + z_2\overline{z}_2$.

(b) (Graduate) Prove that $z_1\overline{z}_2 + \overline{z_1\overline{z}_2} = 2\text{Re}(z_1\overline{z}_2) \leq 2|z_1||z_2|$. NOTE: This corresponds to Schwarz's Inequality from Linear Algebra, which is used in the proof of the Triangle Inequality in $\mathbb{R}^n$.

(c) Use the results in parts (a) and (b) to obtain the inequality $|z_1 + z_2| \leq (|z_1| + |z_2|)^2$, and note how the Triangle Inequality follows.

1.9.2. (b) Show that $\overline{e^{i\theta}} = e^{-i\theta}$.

1.9.6. Show that if $\text{Re}(z_1) > 0$ and $\text{Re}(z_2) > 0$, then $\text{Arg}(z_1z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$.