Complex Variables, MATH 4337/5337, Fall 2020

Homework 5, 2.15. Limits

Due Tuesday, February 25 at 12:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

2.18.2. Let a, b, and c denote complex constants. Use the definition of limit from Section 2.15 to prove the following.

(a) $\lim_{z \to z_0} (az + b) = az_0 + b$.

(b) (Graduate) $\lim_{z \to z_0} (z^2 + c) = z_0 + c$.

2.18.5. (b) Consider the function $f(z) = (z/\overline{z})^2$. Use the definition of limit from Section 2.15 to prove that the limit of $f(z)$ as z tends to 0 does not exist.

2.18.6. (b) Use the definition of limit given in Section 2.15 to prove the first claim in Theorem 2.16.2: If $\lim_{z \to z_0} f(z) = w_0$ and $\lim_{z \to z_0} F(z) = W_0$ then $\lim_{z \to z_0} (f(z) + F(z)) = w_0 + W_0$.

2.18.9. Use the definition of limit given in Section 2.15 to prove that if $\lim_{z \to z_0} f(z) = 0$ and if there exists a positive number M such that $|g(z)| \leq M$ for all z in some neighborhood of z_0, then $\lim_{z \to z_0} f(z)g(z) = 0$.