

Complex Variables, MATH 4337/5337,

Spring 2020

Homework 8, 3.29. The Exponential Function, 3.30. The Logarithm Function,
3.31. Branches and Derivatives of Logarithms

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

3.30.3. Use Theorem 2.21.A to show that the function $f(z) = \exp(\bar{z})$ is not analytic anywhere.

3.30.12. Write $\operatorname{Re}(e^{1/z})$ in terms of x and y . Why is this function harmonic in every domain that does not contain 0? HINT: Show $e^{1/z}$ is analytic in every domain that does not contain 0.

3.30.13. (Graduate) Let $f(z) = u(x, y) + iv(x, y)$ be analytic in some domain D . State why the functions

$$U(x, y) = e^{u(x, y)} \cos v(x, y) \text{ and } V(x, y) = e^{u(x, y)} \sin v(x, y)$$

are harmonic in D .

3.33.3. Show that $\operatorname{Log} i^3 \neq 3\operatorname{Log} i$.

3.33.10. (a) Show that the function $f(z) = \operatorname{Log}(z - i)$ is analytic everywhere except on the portion $x \leq 0$ of the line $y = 1$.