Complex Variables, MATH 4337/5337, Spring 2020

Homework 9, 3.34. Some Identities Involving Logarithms, 3.36. Examples, 4.42. Definite Integrals of Functions $w(t)$; Solutions

Due Thursday, April 16 at 12:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

3.34.3. Choose z_1 and z_2 for which $\operatorname{Log} z_1 z_2 \neq \operatorname{Log} z_1 + \operatorname{Log} z_2$. This shows that Lemma 3.32.A does not hold for the principal branch of the logarithm.

3.36.2. (b) Find the principal value: $\left(\frac{e}{2}(-1-\sqrt{3}i)\right)^{3\pi i}$.

3.33.5. (Graduate) Show that the principal nth root of nonzero complex number z_0 as defined in Section 1.9 (as $c_0 = \sqrt[n]{r}\exp(i\theta_0/n)$ where θ_0 is the principal argument of z_0) is the same as the principal value of $z_0^{1/n}$ defined in this section (as P.V. $z^{1/n} = e^{(1/n)\operatorname{Log} z}$).

4.42.2. (d) Evaluate: $\displaystyle\int_0^\infty e^{-zt}\,dt$ where $\operatorname{Re}(z) > 0$.

4.42.4. According to the definition of a definite integral of a complex-valued function of a real variable,

$$\int_0^\pi e^{(1+i)x}\,dx = \int_0^\pi e^x \cos x\,dx + i\int_0^\pi e^x \sin x\,dx.$$

Evaluate the two integrals on the right here by evaluating the single integral on the left and then by using the real and imaginary parts of the value found.