Complex Variables, MATH 4337/5337, Fall 2024

Homework 1: Sections 1.2. Basic Algebraic Properties, 1.3. Further Properties, 1.5. Complex Conjugates

Due Saturday, January 20 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

1.2.3. Use the properties of complex numbers given in Theorem 1.2.1 to prove that

$$(1+z)^2 = 1 + 2z + z^2.$$

1.2.8(a). Write $(x,y)+(u,v)=(x,y)$ (where this holds for all $(x,y)\in\mathbb{C}$) and prove that the complex number $0=(0,0)$ is unique as an additive identity. You may assume that the additive identity 0 in the real numbers is unique.

1.2.8(b). Write $(x,y)(u,v)=(x,y)$ (where this holds for all $(x,y)\in\mathbb{C}$) and prove that the complex number $1=(1,0)$ is the unique multiplicative identity. You may assume that the multiplicative identity 1 in the real numbers is unique. HINT: Consider two cases. First, suppose $y\neq 0$ and second suppose $y=0$.

1.3.5. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Prove for $z_2 \neq 0$ that

$$\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}.$$

1.5.4. Verify that $\sqrt{2}|z| \geq |\text{Re}(z)| + |\text{Im}(z)|$.

1.5.3. (Graduate Problem) Use established properties of moduli to show that when $|z_3| \neq |z_4|$ we have

$$\frac{\text{Re}(z_1+z_2)}{|z_3+z_4|} \leq \frac{|z_1|+|z_2|}{||z_3|-|z_4||}.$$