Complex Variables, MATH 4337/5337, Fall 2024

Homework 2: Sections 1.5. Triangle Inequality, 1.6. Complex Conjugates, 1.7. Exponential Form, Solutions

Due Saturday, February 3 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

1.5.8. Let z_1 and z_2 denote complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Prove that

$$|(x_1 + iy_1)(x_2 + iy_2)| = \sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}$$

and then prove $|z_1 z_2| = |z_1||z_2|$.

1.6.10. (b) Prove that z is either real or pure imaginary if and only if $\overline{z}^2 = z^2$. HINT: You may assume 1.6.10(a): z is real if and only if $\overline{z} = z$.

1.9.3. Use mathematical induction to prove that $e^{i\theta_1} e^{i\theta_2} \cdots e^{i\theta_n} = e^{i(\theta_1 + \theta_2 + \cdots + \theta_n)}$ for $n = 2, 3, \ldots$.

1.6.15. (Graduate) Follow the steps below to give an algebraic derivation of the Triangle Inequality $|z_1 + z_2| \leq |z_1| + |z_2|$.

(a) Prove that $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = z_1\overline{z}_1 + (z_1\overline{z}_2 + \overline{z_1\overline{z}_2}) + z_2\overline{z}_2$.

(b) Prove that $z_1\overline{z}_2 + \overline{z_1\overline{z}_2} = 2\text{Re}(z_1\overline{z}_2) \leq 2|z_1||z_2|$. NOTE: This corresponds to Schwarz's Inequality from Linear Algebra, which is used in the proof of the Triangle Inequality in $\mathbb{R}^n$.

(c) Use the results in parts (a) and (b) to obtain the inequality $|z_1 + z_2| \leq |z_1| + |z_2|$, and note how the Triangle Inequality follows.