# Complex Variables, MATH 4337/5337, Fall 2024

## Homework 5: Section 13. Mappings, Section 14. Mappings of the Exponential Function, Section 15. Limits

Due Saturday, February 25 at 11:59 pm

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

**2.14.5.** Find a domain in the $z$-plane whose image under the transformation $w = z^2$ is the square domain in the $w$-plane bounded by the lines $u = 1$, $u = 2$, $v = 1$, and $v = 2$.

**2.14.8.** Sketch the region onto which the sector $0 \leq r \leq 1$, $0 \leq \theta \leq \pi/4$ is mapped by the transformations **(a)** $w = z^2$, **(b)** $w = z^3$, **(c)** $w = z^4$.

**2.18.2.** Let $a$, $b$, and $c$ denote complex constants. Use the definition of limit from Section 2.15 to prove the following.

**(a)** $\lim_{z \to z_0} (az + b) = az_0 + b$.

**2.18.2. (Graduate)** Let $a$, $b$, and $c$ denote complex constants. Use the definition of limit from Section 2.15 to prove the following.

**(c)** $\lim_{z \to 1-i} (x + i(2x + y)) = 1 + i$ where $z = x + iy$.

**2.18.2. (Bonus)** Let $a$, $b$, and $c$ denote complex constants. Use the definition of limit from Section 2.15 to prove the following.

**(b)** $\lim_{z \to z_0} (z^2 + c) = z_0^2 + c$.