Complex Variables, MATH 4337/5337, Fall 2024

Homework 6: Section 16. Theorems on Limits, Section 17. Limits Involving the Point at Infinity

Due Saturday, March 2 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

2.18.5. **(a)** Show that the function $f(z) = (z/\overline{z})^2$ has the value 1 at all points on the real and imaginary axes, where $z = (x, 0)$ and $z = (0, y)$ respectively, but that it has the value -1 at all nonzero points on the line $y = x$, where $z = (x, x)$. (Notice that this can be used to show that the limit of $f(z)$ as z tends to 0 does not exist, as is to be done in 2.18.5(b).)

2.18.6. **(b)** Prove the first claim in Theorem 2.16.2: If $\lim_{z\to z_0} f(z) = w_0$ and $\lim_{z\to z_0} F(z) = W_0$ then $\lim_{z\to z_0}(f(z)+F(z)) = w_0+W_0$. Use the definition of limit given in Section 2.15. HINT: This is an $\varepsilon/2$ proof.

2.18.10. Use Theorem 2.17.1 to show the following.

(a) $\lim\limits_{z\to\infty} \dfrac{4z^2}{(z-1)^2} = 4.$

(b) $\lim\limits_{z\to 1} \dfrac{1}{(z-1)^3} = \infty.$