

Complex Variables, MATH 4337/5337, Fall 2024

Homework 9: Section 31. Branches and Derivatives of Logarithms, Section 32.

Some Identities Involving Logarithms

Due Saturday, April 13 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. The exercise numbers are based on the 9th edition of the textbook.

3.33.4. Show that $\log i^2 \neq 2 \log i$ when the branch $\log z = \ln r + i\theta$ where $z = re^{i\theta}$, $r = |z| > 0$, and $\theta \in (3\pi/4, 11\pi/4)$ is used.

3.33.10. (a) Show that the function $f(z) = \operatorname{Log}(z-i)$ is analytic everywhere except on the portion $x \leq 0$ of the line $y = 1$.

3.34.2. Prove that $\log(z_1/z_2) = \log z_1 - \log z_2$ as follows.

(a) Use the fact that $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$ by Lemma 1.8.2.

3.34.2. (Graduate) Prove that $\log(z_1/z_2) = \log z_1 - \log z_2$ as follows.

(b) Show that $\log 1/z = -\log z$ for $z \neq 0$ in the sense that $\log 1/z$ and $-\log z$ have the same set of values, and then refer to Lemma 3.32.A.