

Complex Variables, MATH 4337, Spring 2025

Homework 1: Sections 1.2. Basic Algebraic Properties,

1.3. Further Properties

Due Saturday, January 25 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). The exercise numbers are based on the 9th edition of the textbook.

1.2.7. Use the Associative Law for Addition and the Distributive Law to prove that

$$z(z_1 + z_2 + z_3) = zz_1 + zz_2 + zz_3.$$

HINT: Notice by Associativity of Addition (Theorem 1.2.1(2)) we have that $z_1 + z_2 + z_3$ represents both $(z_1 + z_2) + z_3$ and $z_1 + (z_2 + z_3)$.

1.2.11. Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$ by writing

$$(x, y)(x, y) + (x, y) + (1, 0) = (0, 0)$$

and then solving a pair of simultaneous equations in x and y . **Write in complete sentences and explain** your reasoning on this one!

1.3.3. Use the Associative and Commutative Laws for Multiplication to prove that $(z_1z_2)(z_3z_4) = (z_1z_3)(z_2z_4)$.

1.3.8. (Graduate) Use mathematical induction to verify the binomial formula:

$$(z_1 + z_2)^n = \sum_{k=0}^n \binom{n}{k} z_1^k z_2^{n-k} \text{ for } n \in \mathbb{N}.$$