

Complex Variables, MATH 4337, Spring 2025

Homework 10: Section 40. Contour Integrals, Section 41. Some

Examples, Section 42. Examples with Branch Cuts

Due Saturday, April 19 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). The exercise numbers are based on the 9th edition of the textbook.

4.43.1. If $w(t) = u(t) + iv(t)$ is continuous on an interval $t \in [a, b]$, then show the following.

(b) $\int_a^b w(t) dt = \int_\alpha^\beta w(\varphi(\tau))\varphi'(\tau) d\tau$ where $t = \varphi(\tau)$ is a real-valued function with continuous derivative satisfying $\varphi'(\tau) > 0$ which maps the interval $\tau \in [\alpha, \beta]$ onto the interval $t \in [a, b]$ (so that $\varphi(\alpha) = a$ and $\varphi(\beta) = b$).

4.43.4. Suppose contour C is represented as $z = Z(\tau) = z(\varphi(\tau))$ where $\tau \in [\alpha, \beta]$ and $t = \varphi(\tau)$ (where $t \in [a, b]$) is a real-valued function with continuous derivative satisfying $\varphi'(\tau) > 0$ which maps the interval $\tau \in [\alpha, \beta]$ onto the interval $t \in [a, b]$. Show that $Z'(\tau) = z'(\varphi(\tau))\varphi'(\tau)$.

4.46.1. Use a parametric representation for C to evaluate $\int_C f(z) dz$ for $f(z) = (z + 2)/z$ and C as follows.

(c) C is the circle $z(\theta) = 2e^{i\theta}$ where $\theta \in [0, 2\pi]$.