Complex Variables, MATH 4337, Spring 2025

Homework 11: Section 42. Examples with Branch Cut, Section 43. Upper Bounds for Moduli of Contour Integrals, Solutions

Due Saturday, April 26 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`). The exercise numbers are based on the 9th edition of the textbook.

4.46.11. Let C denote the positively oriented right-hand half of the circle $|z| = 2$. Evaluate the integral of the function $f(z) = \overline{z}$ along C using the given parametric representation.

(a) $z(\theta) = 2e^{i\theta}$ where $\theta \in [-\pi/2, \pi/2]$.

4.46.13. Let C_0 denote the circle centered at z_0 with radius R, and use the parametrization $z = z_0 + Re^{i\theta}$ where $\theta \in [-\pi, \pi]$ to show that

$$\int_{C_0} (z - z_0)^{n-1}\, dz = \begin{cases} 0 & \text{when } n \in \mathbb{Z} \setminus \{0\} \\ 2\pi i & \text{when } n = 0. \end{cases}$$

4.47.2. Let C denote the line segment from $z = i$ to $z = -1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin (you may assume this geometric property), show that $\left| \int_C \frac{dz}{z^4} \right| \leq 4\sqrt{2}$ without evaluating the integral.