Complex Variables, MATH 4337, Spring 2025

Homework 2: Sections 1.5. Triangle Inequality, 1.6. Complex Conjugates, 1.7. Exponential Form

Due Saturday, February 1 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`). The exercise numbers are based on the 9th edition of the textbook.

1.5.3. Use established properties of moduli to show that when $|z_3| \neq |z_4|$ we have

$$\frac{\text{Re}(z_1 + z_2)}{|z_3 + z_4|} \leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||}.$$

1.6.13. Prove that the equation $|z - z_0| = R$ of a circle centered at z_0 with radius R, can be written as $|z|^2 - 2\text{Re}(z\overline{z}_0) + |z_0|^2 = R^2$.

1.9.1(a), 1.9.2(a)

1.9.1(a) Find the principal argument $\text{Arg}(z)$ where: $z = \dfrac{-2}{1 + \sqrt{3}i}$.

1.9.2(a) Show that $|e^{i\theta}| = 1$.

1.6.15. (Graduate) Follow the steps below to give an algebraic derivation of the Triangle Inequality $|z_1 + z_2| \leq |z_1| + |z_2|$. Do parts (b) and (c). You may assume part (a)

(a) Prove that $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = z_1\overline{z}_1 + (z_1\overline{z}_2 + \overline{z_1\overline{z}_2}) + z_2\overline{z}_2$.

(b) Prove that $z_1\overline{z}_2 + \overline{z_1\overline{z}_2} = 2\text{Re}(z_1\overline{z}_2) \leq 2|z_1||z_2|$. NOTE: This corresponds to Schwarz's Inequality from Linear Algebra, which is used in the proof of the Triangle Inequality in $\mathbb{R}^n$.

(c) Use the results in parts (a) and (b) to obtain the inequality $|z_1 + z_2| \leq |z_1| + |z_2|$, and note how the Triangle Inequality follows.